Mid-term Examination – Fall 2024
Time: 24 hours (Nov 7-8, 2024)

1. Calculate the integral

$$I(\sigma) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{\infty} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \cos x \, dx$$

2. Calculate

$$S = \sum_{n=0}^{\infty} e^{-\alpha n}; \qquad n = 0, 1, 2, \ldots$$

3. Show that for a space of large dimension, $d \gg 1$, the volume of a hyper-sphere of radius R mostly resides in the surface.

4. Assuming

$$\delta(\sin x - \cos x) = \sum_n c_n \delta(x - x_n),$$

obtain the values of c_n and x_n.

5. (a) The specific heat at constant volume, C_v, is given by

$$C_v = \left(\frac{\partial U}{\partial T}\right)_v.$$

Express C_v in terms of (the derivatives of) $\ln Z_c$, where Z_c is the canonical partition function.

(b) The energy levels of a two-level system are given by $\pm\Delta$. Derive the canonical partition function Z_c and calculate C_v for the system.

(c) Sketch C_v as a function of the reduced temperature $K_b T/\Delta$.

6. Consider a single particle in one dimension in a harmonic potential

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2.$$

Calculate the partition function, $\Omega(E)$, in microcanonical ensembles.