Practice Problems: I

1. Prove the Stirling formula
$$x! \approx \sqrt{2\pi x}\, x^x \;\exp(-x)$$

2. Given
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-x_0)^2}{2\sigma^2}}; \qquad x \in [-\infty, \infty]$$
Calculate $\langle x \rangle$, $\langle x^2 \rangle$.

3. Let the statistical variable X has the property that $\ln X$ obeys a Gaussian distribution with $\langle \ln X \rangle = \ln x_0$

   a) Show that the probability distribution of X is given by,
$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{x} \ln(x/x_0)^2 \exp\left(-\frac{1}{\sigma}\right)$$

   b) Show that
$$\langle x \rangle = x_0 e^{\sigma^2/2}$$
   and
$$\langle \ln x \rangle = \ln x_0$$

4. Given
$$S = \frac{M!}{N!\,(M-N)!}$$
Find the value of N for which S is maximum.

5. Prove that
$$x\delta'(x) = -\delta(x)$$
and
$$\delta(ax) = \frac{1}{|a|}\delta(x)$$

6. Evaluate
$$I(\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{(x-x_0)^2}{2\sigma^2}}\, dx$$

7. Find the volume of a sphere of unit radius in d dimensions. Use the results to plot the volume as a function of dimension d.

8. Show that

$$\lim_{x \to 0^+} \Gamma(x) = \infty$$

and plot $\Gamma(x)$ as a function of x, where x ranges from 0 to a large positive number.

9. Evaluate

$$I = \int_0^2 x^3 (4 - x^2)^{3/2}\, dx$$

10. Evaluate

$$I = \int_0^\infty \frac{dx}{1 + u^4}$$