Problem - 0 (Spring 2024)

Time allotted: Two weeks

1. The error function is defined as

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

Show that the derivative of $erf(x)$ is given by

$$\frac{d}{dx}\big[erf(x)\big] = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

2. a) Consider the integrals below

$$X(s) = \int_0^s dt\, \cos\left(\frac{1}{2}\pi t^2\right)$$

and

$$Y(s) = \int_0^s dt\, \sin\left(\frac{1}{2}\pi t^2\right).$$

For a given value of s, say $s = 10$, plot $Y(s)$ vs. $X(s)$. You may use MatLab/Python functions to obtain the value of the integrals for the purpose of plotting.

b) Verify analytically or numerically that for $s \to \infty$, the integrals approach to 1/2.

3. Plot the following functions:

a) $\Theta(1 - x^2)$ vs. x

b) $x\Theta(\sin(x))$ vs. x

c) $\Theta(tanh(x))$

4. Prove that

$$\delta(g(x)) = \sum_{i=1}^{n} \frac{\delta(x - x_i)}{|g'(x)|},$$

where x_i are the zeroes of the function $g(x)$.